Helping Book
*series
Integral calculus
by
Krishna Prakashan

Multiple Integrals

in this

Change of Order of Integration

for B.Sc.- I

By

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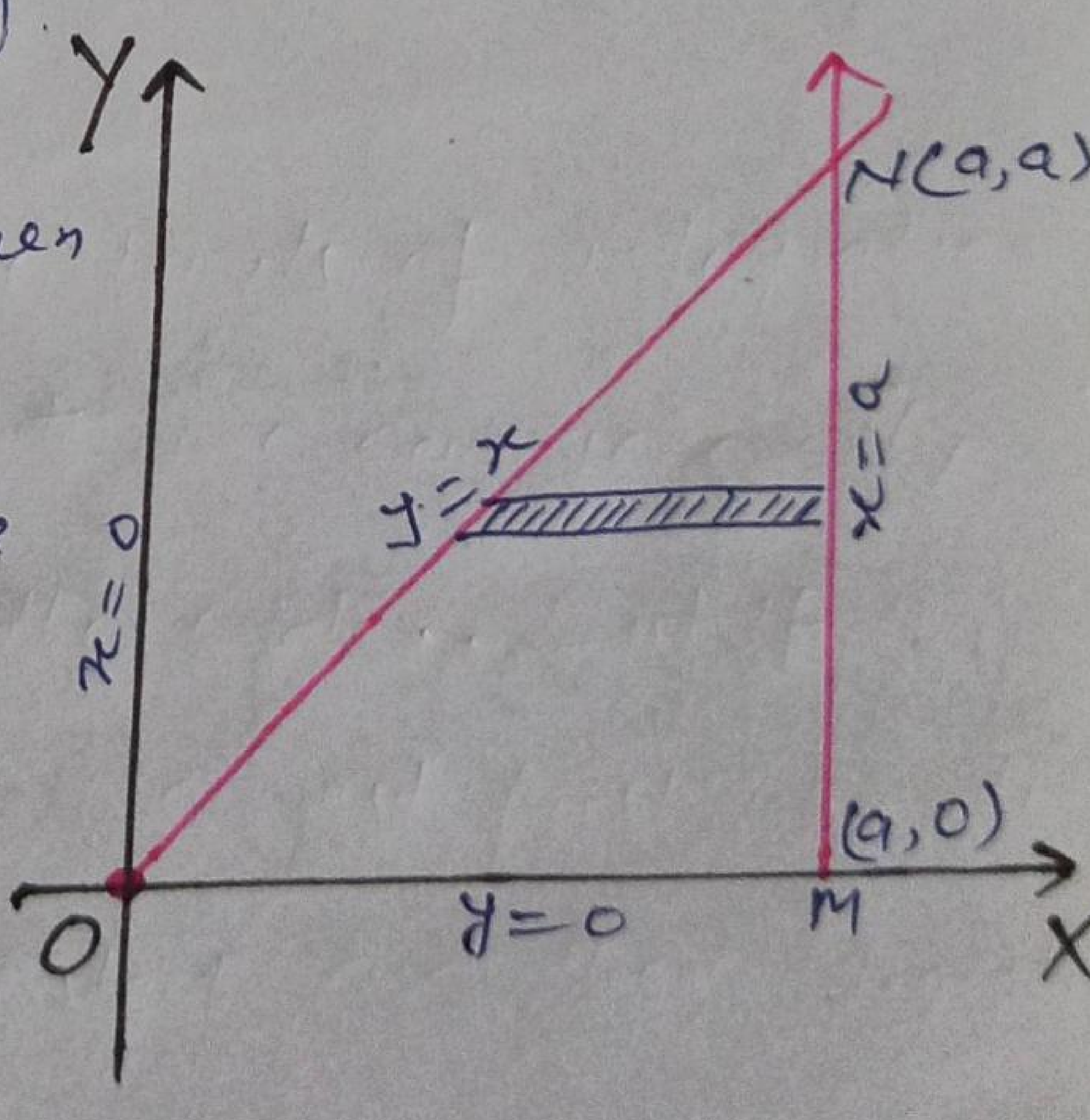
$\Rightarrow$ If in a double integral the limits of integration of both x and y are constant, we can generally integrate $\iint f(x,y)\,dx\,dy$ in either order. But if the limits of y are functions of x, we must first integrate w.r.to y regarding x as constant and then integrate w.r.t. x

In this case the order of integration can be changed only if we find the new limits of x as functions of y and the new constant limit of y. This usually best obtained from geometrical considerations as will be clear from the examples that follow.

Ex. ① Change the order of integration in the double integral $\int_0^a \int_0^x f(x,y)\,dx\,dy$

Sol^n. In the given integral the limits of integration are given by the straight lines $y=0$, $y=x$, $x=0$, $x=a$. We draw these lines bounding the region of integration in the figure.



In the given integral, the limits of integration of y being variable, we are required to integrate first w.r.t. y regarding x as constant and then w.r.t. x.

To reverse the order of integration, we have to integrate first w.r.t. x regarding y as constant and then w.r.to y. This is done by dividing the area ONM into strips parallel to the x-axis.

Let us take strips parallel to the x-axis starting from the line ON (i.e., $y=x$) and terminating on the line MN (i.e., $x=a$). Thus for this region ONM, x varies from y to a and y varies from 0 to a.

Hence by changing the order of integration, we get

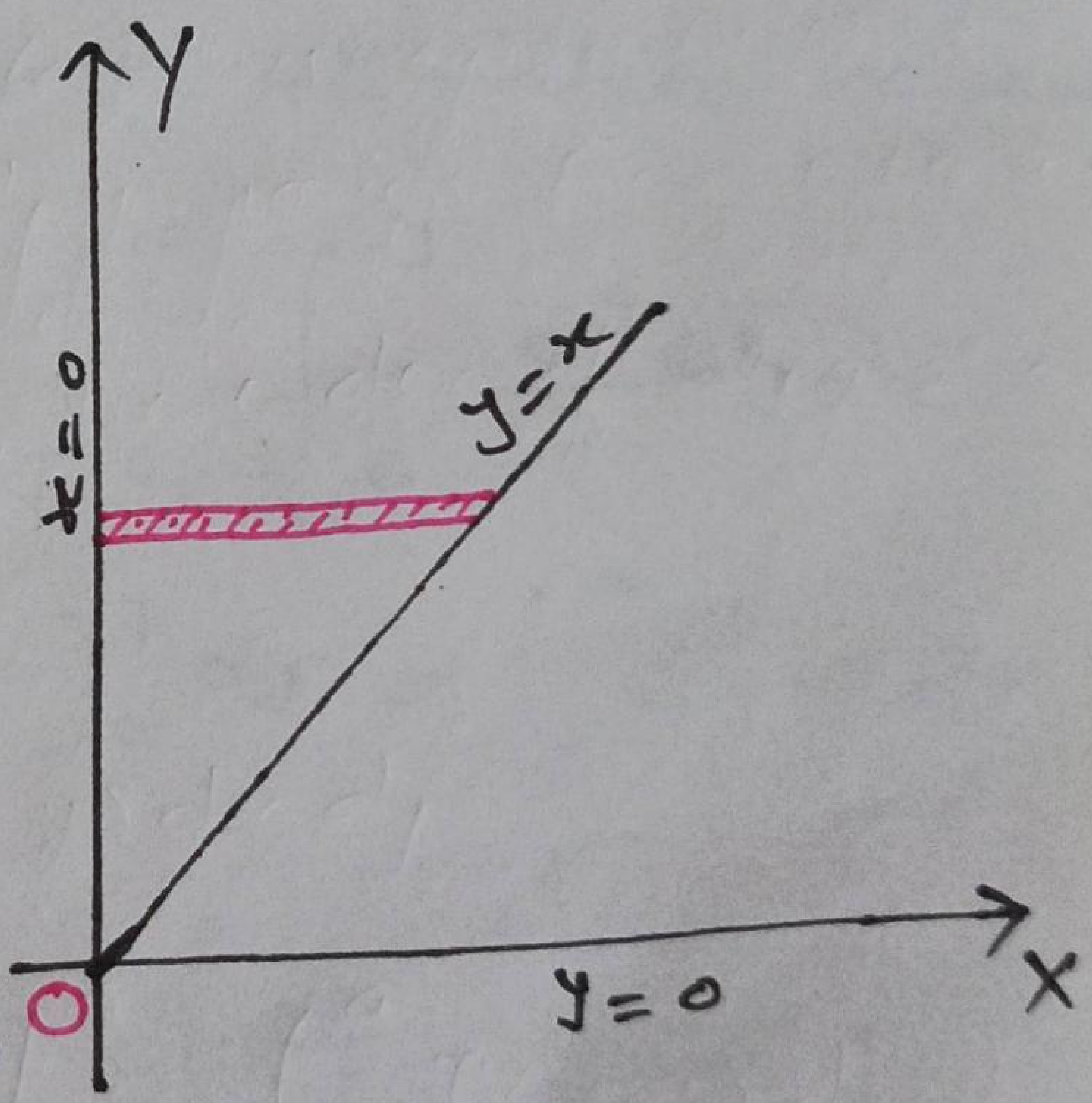
$$\int_0^a \int_0^x f(x,y)\,dx\,dy$$

$$= \int_0^a \int_y^a f(x,y)\,dy\,dx$$

Ex. 2. Imp. Change the order of integration in the double integral

$$\int_0^{\infty}\int_x^{\infty} \frac{e^{-y}}{y}\,dx\,dy$$ and hence find the value.

Sol.ⁿ In the given integral the limits of integration are given by the lines $y = x$, $y = \infty$, $x = 0$ and $x = \infty$. Therefore the region of integration is bounded by $x = 0$, $y = x$ and an infinite boundary. Thus we have to first integrate with respect to y regarding x as constant and then integrate w.r.to x.



To reverse the order of integration, we have to first integrate w.r.t. x regarding y as constant and then integrate w.r.to y.

This is done by dividing this area into strips parallel to the x-axis. So we take strips parallel to the x-axis starting from the line $x=0$ and terminating on the line $y=x$. Now the limits for x are 0 to y and the limits for y are 0 to ∞.

Hence by changing the order of integration, we have

$$\int_{x=0}^{\infty}\int_{y=x}^{\infty}\frac{e^{-y}}{y}\,dx\,dy = \int_{y=0}^{\infty}\int_{x=0}^{y}\frac{e^{-y}}{y}\,dy\,dx$$

$$= \int_0^{\infty}\frac{e^{-y}}{y}\left[x\right]_0^y dy = \int_0^{\infty}\frac{e^{-y}}{y}\cdot y\,dy$$

$$= \int_0^{\infty} e^{-y}\,dy = \left[\frac{e^{-y}}{-1}\right]_0^{\infty} = 1$$