MATRICES

FOR B.Sc. II

Helping Book -
Linear Algebra
&
Matrices
by Krishnaprakashan.

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⇒ **Matrix**

Definition A set of mn numbers (real or complex) arranged in the form of a rectangular array having m rows and n columns is called an $m \times n$ matrix (to be read as 'm by n matrix').

An $m \times n$ matrix is written as

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} & \cdots & a_{1n} \\ a_{21} & a_{22} & a_{23} & \cdots & a_{2n} \\ a_{31} & a_{32} & a_{33} & \cdots & a_{3n} \\ \vdots & \vdots & \vdots & & \vdots \\ a_{m1} & a_{m2} & a_{m3} & \cdots & a_{mn} \end{bmatrix}$$

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Note → The element a_{ij} belong to the i^{th} row and j^{th} column and is called the $(i,j)^{th}$ element of the matrix.

Special Types of Matrices -

(i) Square Matrix - An $m \times n$ matrix for which $m=n$ (i.e., the number of rows is equal to the number of column) is called a square matrix of order n.

For ex. $A = \begin{bmatrix} 1 & 3 \\ 4 & 7 \end{bmatrix}_{2\times2}$ or $A = \begin{bmatrix} 1 & 2 & 3 \\ 7 & 5 & 1 \\ 2 & 0 & 5 \end{bmatrix}_{3\times3}$

Note → The elements a_{ij} of a square matrix $A=[a_{ij}]_{n\times n}$ for which $i=j$ i.e., the elements $a_{11}, a_{22}, a_{33} \cdots$ $\cdots a_{nn}$ are called the diagonal elements and the line along which they lie is called the principal diagonal of the matrix. For Example, the matrix

$$A = \begin{bmatrix} 2 & 3 & 5 \\ 4 & 4 & 1 \\ 7 & 6 & 3 \end{bmatrix}_{3\times3}$$

is a square matrix of order 3. The elements 2, 4, 3 constitute the principal diagonal of this matrix.

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2. Unit Matrix or Identity Matrix -

A square matrix each of whose diagonal elements is 1 and each of whose non-diagonal elements is equal to zero is called a unit matrix or an identity matrix and is denoted by I.

I_n will denote a unit matrix of order n. Thus a square matrix $A = [a_{ij}]$ is a unit matrix if $a_{ij} = 1$ when $i = j$ and $a_{ij} = 0$ when $i \neq j$.

For Example

$$I_2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \quad I_3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

3. Null Matrix or Zero Matrix -

The $m \times n$ matrix whose elements are all 0 is called the null matrix (or zero matrix) of the type $m \times n$. It is usually denoted by O.

for example

$$\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}_{4\times 3}, \text{ and } \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}_{2\times 2}$$

are zero matrices of the order 4×3 and 2×2 respectively.

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4. Row Matrix and Column Matrices

Any $1 \times n$ matrix which has only one row and n columns is called a row matrix ~~and~~ or row vector. Similarly any $m \times 1$ matrix has m rows and only one column is a column matrix or a column vector.

For Example

$$A = \begin{bmatrix} 3 & 7 & 2 & -8 & 5 \end{bmatrix}_{1 \times 5}$$ is a row matrix

of type 1×5. and

$$B = \begin{bmatrix} 3 \\ 2 \\ -8 \\ 1 \end{bmatrix}_{4 \times}$$ is a column matrix of the type 4×1.

⇒ **Submatrices of a Matrix** - Any matrix obtained by omitting some rows and columns from a given $m \times n$ matrix A is called a <u>Submatrix</u> of A.

<u>Note</u> - A square submatrix of a square matrix A is called a principal submatrix, if its diagonal elements are also the diagonal elements of the matrix A. Principal submatrices are obtained only by omitting corresponding rows and columns.

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<u>For Example</u>

The Matrix $\begin{bmatrix} 1 & 2 & 3 \\ 0 & 2 & 1 \end{bmatrix}$ is a submatrix of the matrix $A = \begin{bmatrix} 1 & 2 & 3 & 9 \\ 7 & 11 & 6 & 5 \\ 0 & 2 & 1 & 2 \end{bmatrix}$

as it can be obtained from A by omitting the second row and the fourth column.

⇒ Equality of Two Matrices

Two Matrices $A = [a_{ij}]$ and $B = [b_{ij}]$ are said to be equal, if

(i) they are of the same size and

(ii) the elements in the corresponding places of the two matrices are the same i.e., $a_{ij} = b_{ij}$ for each pair of subscript i and j.

<u>For Example</u> $A = \begin{bmatrix} 2 & 3 \\ 0 & 5 \end{bmatrix}_{2\times 2}$. $B = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$

If $A = B$ then $a = 2$, $b = 3$, $c = 0$ and $d = 5$.

to be Cont......

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