Helping Book
Linear Algebra
&
Matrices
by
Krishna prakashan

Matrices

II Part for B.Sc-II

By.

Jitendra Kumar
Asstt. Prof. (Mathematics)
GDC Bhojpur, Moradabad

⇒ Addition of Matrices

Let A and B be two matrics of the same type $m \times n$. Then their sum (to be denoted by $A+B$) is defined to be the matrix of the type $m \times n$ obtained by adding the corresponding elements of A and B.

Thus if $A = [a_{ij}]_{m\times n}$ and $B = [b_{ij}]_{m\times n}$

then $A+B = [a_{ij} + b_{ij}]_{m\times n}$.

For example if

$$A = \begin{bmatrix} 2 & 3 & -1 \\ -3 & 4 & 2 \end{bmatrix}_{2\times 3} \text{ and } B = \begin{bmatrix} 2 & -2 & 6 \\ 4 & 2 & -1 \end{bmatrix}_{2\times 3}$$

then $$A+B = \begin{bmatrix} 2+2 & 3-2 & -1+6 \\ -3+4 & 4+2 & 2-1 \end{bmatrix} = \begin{bmatrix} 4 & 1 & 5 \\ 1 & 6 & 1 \end{bmatrix}_{2\times 3}$$

Note - It should be noted that addition is defined only for matrices which are of the same size. If two matrices A and B are of the same size, they are said to be conformable for addition. If the matrices A and B are not of the same size, we cannot find their sum.

⇒ Multiplication of a Matrix by a Scalar

Let A be any $m \times n$ matrix and K any complex number called scalar. Then $m \times n$ matrix obtained by multiplying every element of the matrix A by K is called the scalar multiple of A by K and is denoted by KA OR AK. Symbolically

$$\text{If } A = [a_{ij}]_{m \times n} \text{ then } KA = AK = [ka_{ij}]_{m \times n}$$

For example if $K = 3$ and $A = \begin{bmatrix} 3 & -1 \\ 4 & 2 \end{bmatrix}_{2 \times 2}$

then $3A = \begin{bmatrix} 9 & -3 \\ 12 & 6 \end{bmatrix}$

Multiplication of Two Matrices

Let $A = [a_{ij}]_{m \times n}$ and $B = [b_{jk}]_{n \times p}$ be two matrices such that the number of column in A is equal to the number of rows in B. Then the $m \times p$ matrix

$C = [c_{ik}]_{m \times p}$ such that

$$c_{ik} = \sum_{j=1}^{n} a_{ij} b_{jk}$$ [Note that the summation is with respect to the repeated suffix j]

is called the product of the matrices A and B in that order and we write $C = AB$.

for example, if $A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \\ a_{31} & a_{32} \end{bmatrix}_{3 \times 2}$, $B = \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix}_{2 \times 2}$

then $AB = \begin{bmatrix} a_{11}b_{11} + a_{12}b_{21} & a_{11}b_{12} + a_{12}b_{22} \\ a_{21}b_{11} + a_{22}b_{21} & a_{21}b_{12} + a_{22}b_{22} \\ a_{31}b_{11} + a_{32}b_{21} & a_{31}b_{12} + a_{32}b_{22} \end{bmatrix}_{3 \times 2}$

Note If the product AB exists, then it is not necessary that the product BA will also exist. For example, If A is a 4×5 matrix and B is a 5×3 matrix, then the product AB exists while the BA does not exist.

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Triangular, Diagonal and Scalar Matrics

(i) Upper Triangular Matrix - A square matrix $A = [a_{ij}]$ is called an upper triangular matrix if $a_{ij} = 0$ whenever $i > j$.

Thus in an upper triangular matrix all the elements below the principal diagonal are zero.

for Example

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ 0 & a_{22} & a_{23} & a_{24} \\ 0 & 0 & a_{33} & a_{34} \\ 0 & 0 & 0 & a_{44} \end{bmatrix}.$$

(ii) Lower Triangular Matrix

A square matrix $A = [a_{ij}]$ is called a lower triangular matrix if $a_{ij} = 0$ whenever $i < j$. Thus in a lower triangular matrix all the elements above the principal diagonal are zero.

for example
$$A = \begin{bmatrix} a_{11} & 0 & 0 \\ a_{21} & a_{22} & 0 \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$$

(iii) Diagonal Matrix

A square matrix $A = [a_{ij}]_{n \times n}$ whose elements above and below the principal diagonal are all zero, i.e, $a_{ij} = 0$ for all $i \neq j$, is called a diagonal matrix.

(iv) Scalar matrix

A diagonal matrix whose diagonal elements are all equal is called a scalar matrix.

$$A = \begin{bmatrix} K & 0 & 0 \\ 0 & K & 0 \\ 0 & 0 & K \end{bmatrix}$$

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⇒ Idempotent, Involutary, Nilpotent and Periodic Matrices

Idempotent Matrix

A matrix such that $A^2 = A$ is called idempotent matrix.

Involutary Matrix

A matrix A is said to be an involutary matrix if $A^2 = I$ (unit matrix)

Note $I^2 = I$, we conclude that a unit matrix is always involutory.

Nilpotent Matrix A matrix is said to be a nilpotent matrix if $A^k = 0$ (null matrix) where k is a +ve integer.

for example $A = \begin{bmatrix} ab & b^2 \\ -a^2 & -ab \end{bmatrix}$ is nilpotent of index 2.

Periodic Matrix. A matrix A is said to be a periodic matrix if $A^{k+1} = A$, where k is a +ve integer.

If k is the least +ve integer for which $A^{k+1} = A$ then k is said to be period of A.

⇒ Determinant of a Square Matrix

Let A be any square matrix. The determinant formed by the elements of A is said to be the determinant of matrix A. This is denoted by $|A|$ or $\det A$.

Ex. If $A = \begin{bmatrix} 3 & 2 & 7 \\ 5 & 1 & 4 \\ 2 & 0 & 8 \end{bmatrix}$, then $\det A = |A| = \begin{vmatrix} 3 & 2 & 7 \\ 5 & 1 & 4 \\ 2 & 0 & 8 \end{vmatrix}$

Difference between a matrix and a determinant

1. A matrix 'A' cannot be reduced to a number whereas the determinant can be reduced to a number.
2. The number of rows may or may not be equal to number of columns in a matrix while in a determinant the number of rows is equal to the number of columns.

3. Interchanging the rows and columns, a different Matrix is formed while in determinant, an interchange of rows and columns does not change the value of the determinant.

Non-Singular and Singular Matrices

A square matrix A is said to be non-singular or singular according as

$|A| \neq 0$ or $|A| = 0$

To be cont. ...