

Multiple Integrals

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Change of Order of Integration

For B.Sc. I

By

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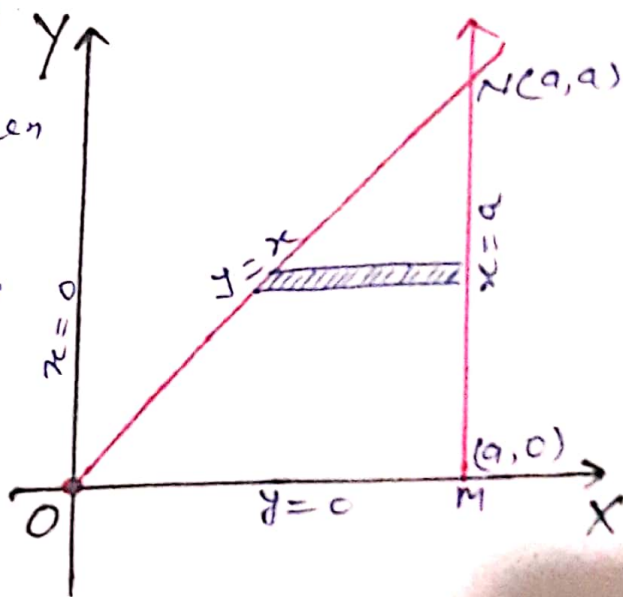
⇒ If in a double integral the limits of integration of both x and y are constant, we can generally integrate $\iint f(x,y) dx dy$ in either order. But if the limits of y are functions of x , we must first integrate w.r.t. y ~~regarding~~ regarding x as constant and then integrate w.r.t. x .

In this case the order of integration can be changed only if we find the new limits of x as functions of y and the new constant limit of y . This usually best obtained from geometrical considerations as will be clear from the examples that follow.

Ex. ① Change the order of integration in the double integral $\int_0^a \int_0^x f(x, y) dx dy$

Solⁿ.

In the given integral the limits of integration are given by the straight lines $y=0$, $y=x$, $x=0$, $x=a$. we draw these lines bounding the region of integration in the figure.



In the given integral, the limits of integration of y being variable, we are required to integrate first w.r.t. y regarding x as constant and then w.r.t. x .

To reverse the order of integration, we have to integrate first w.r.t. x regarding y as constant and then w.r.t. y . This is done by dividing the area ONM into strips parallel to the x -axis.

Let us take strips parallel to the x -axis starting from the line ON (i.e., $y = x$) and terminating on the line MN (i.e., $x = a$). Thus for this region ONM , x varies from y to a and y varies from 0 to a .

Hence by changing the order of integration, we get

$$\int_0^a \int_0^x f(x, y) dx dy$$

$$= \int_0^a \int_y^a f(x, y) dy dx$$

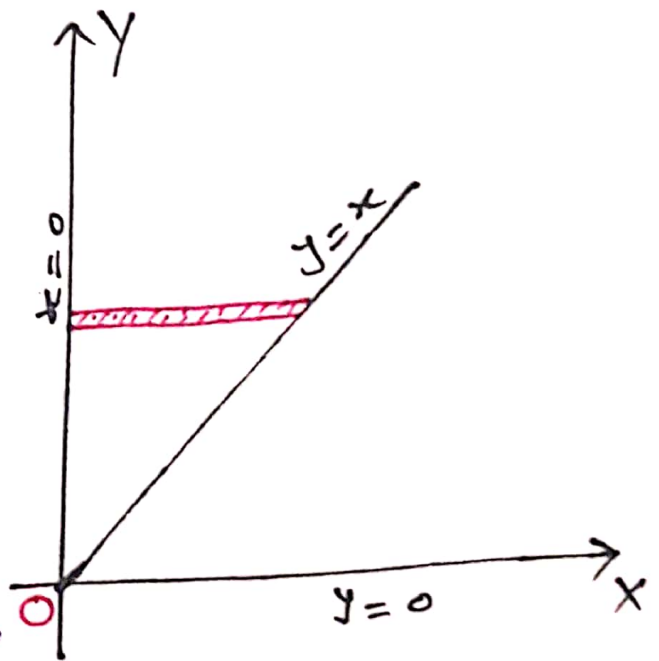
Ex. 2. Change the order of integration in the double integral

$$\int_0^{\infty} \int_x^{\infty} \frac{e^{-y}}{y} dx dy \text{ and hence}$$

find the value.

Sol: In the given integral

the limits of integration are given by the lines $y=x$, $y=\infty$, $x=0$ and $x=\infty$. Therefore the region of integration is bounded by $x=0$, $y=x$ and an infinite boundary. Thus we have to first integrate with respect to y regarding x as constant and then integrate w.r.t. x .



To reverse the order of integration, we have to first integrate w.r.t. x regarding y as constant and then integrate w.r.t. y .

This is done by dividing this area into strips parallel to the x -axis. So we take strips parallel to the x -axis starting from the line $x=0$ and terminating on the line $y=x$. Now the limits for x are 0 to y and the limits for y are 0 to ∞ .

Hence by changing the order of integration, we have

$$\begin{aligned}\int_0^{\infty} \int_x^{\infty} \frac{e^{-y}}{y} dx dy &= \int_0^{\infty} \int_0^y \frac{e^{-y}}{y} dy dx \\ &= \int_0^{\infty} \frac{e^{-y}}{y} [x]_0^y dy = \int_0^{\infty} \frac{e^{-y}}{y} \cdot y dy \\ &= \int_0^{\infty} e^{-y} dy = \left[\frac{e^{-y}}{-1} \right]_0^{\infty} = 1\end{aligned}$$