

MATRICES

FOR B.Sc. II

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⇒ Matrix

Definition

A set of $m \times n$ numbers (real or complex) arranged in the form of a rectangular array having m rows and n columns is called an $m \times n$ matrix (to be read as 'm by n matrix').

An $m \times n$ matrix is written as

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} & \dots & a_{1n} \\ a_{21} & a_{22} & a_{23} & \dots & a_{2n} \\ a_{31} & a_{32} & a_{33} & \dots & a_{3n} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ a_{m1} & a_{m2} & a_{m3} & \dots & a_{mn} \end{bmatrix}$$

Note → The element a_{ij} belong to the i^{th} row and j^{th} column and is called the $(i,j)^{\text{th}}$ element of the matrix.

Special Types of Matrices -

(i) Square Matrix - An $m \times n$ matrix for which $m=n$ (i.e., the number of rows is equal to the number of column) is called a square matrix of order n .

For ex. $A = \begin{bmatrix} 1 & 3 \\ 4 & 7 \end{bmatrix}_{2 \times 2}$ or $A = \begin{bmatrix} 1 & 2 & 3 \\ 7 & 5 & 1 \\ 2 & 0 & 5 \end{bmatrix}_{3 \times 3}$

Note → The elements a_{ij} of a square matrix $A = [a_{ij}]_{n \times n}$ for which $i=j$ i.e., the elements $a_{11}, a_{22}, a_{33}, \dots, a_{nn}$ are called the diagonal elements and the line along which they lie is called the principal diagonal of the matrix. For example, the matrix

$$A = \begin{bmatrix} 2 & 3 & 5 \\ 7 & 4 & 1 \\ 7 & 6 & 3 \end{bmatrix}_{3 \times 3}$$

is a square matrix of order 3. The elements 2, 4, 3 constitute the principal diagonal of this matrix.

2. Unit Matrix or Identity Matrix -

A square matrix each of whose diagonal elements is 1 and each of whose non-diagonal elements is equal to zero is called a unit matrix or an identity matrix and is denoted by I .

I_n will denote a unit matrix of order n . Thus a square matrix $A = [a_{ij}]$ is a unit matrix if $a_{ij} = 1$ when $i = j$ and $a_{ij} = 0$ when $i \neq j$.

For example

$$I_2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \quad I_3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

3. Null Matrix or Zero Matrix -

The $m \times n$ matrix whose elements are all 0 is called the null matrix (or zero matrix) of the type $m \times n$. It is usually denoted by O .

For example

$$\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}_{4 \times 3}, \text{ and } \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}_{2 \times 2}$$

are zero matrices of the order 4×3 and 2×2 respectively.

4. Row Matrix and Column Matrices

Any $1 \times n$ matrix which has only one row and n columns is called a row matrix ~~and~~ or row vector. Similarly any $m \times 1$ matrix has m rows and only one column is a column matrix or a column vector.

For Example

$A = [3 \ 7 \ 2 \ -8 \ 5]_{1 \times 5}$ is a row matrix of type 1×5 . and

$B = \begin{bmatrix} 3 \\ 2 \\ -8 \\ 1 \end{bmatrix}_{4 \times 1}$ is a column matrix of the type 4×1 .

\Rightarrow Submatrices of a Matrix - Any matrix obtained by omitting some rows and columns from a given $m \times n$ matrix A is called a Submatrix of A .

Note - A square submatrix of a square matrix A is called a principal submatrix, if its diagonal elements are also the diagonal elements of the matrix A . Principal submatrices are obtained only by omitting corresponding rows and columns.

For Example

The matrix $\begin{bmatrix} 1 & 2 & 3 \\ 0 & 2 & 1 \end{bmatrix}$ is a submatrix of the matrix $A = \begin{bmatrix} 1 & 2 & 3 & 9 \\ 7 & 11 & 6 & 5 \\ 0 & 2 & 1 & 8 \end{bmatrix}$

as it can be obtained from A by omitting the second row and the fourth column.

⇒ Equality of Two Matrices

Two matrices $A = [a_{ij}]$ and $B = [b_{ij}]$ are said to be equal, if

- (i) they are of the same size and
- (ii) the elements in the corresponding places of the two matrices are the same i.e., $a_{ij} = b_{ij}$ for each pair of subscript i and j .

For Example $A = \begin{bmatrix} 2 & 3 \\ 0 & 5 \end{bmatrix}_{2 \times 2}$. $B = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$

If $A=B$ then $a=2, b=3, c=0$ and $d=5$.
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