Electrostatics

for B.Sc. (II) Year

Paper - II

Prepared By:
Dr. Manoj Kumar Paras
Assistant Prof. - Physics
Govt. Degree college Bhojpur
Moradabad.

⇒ When two bodies are rubbed with each other they get charged.

⇒ **Types of Charges:**

1. Positive charge
2. Negative charge

Unit of charge is coulomb (C).

⇒ **Conservation of charges:** Charge can neither be created nor destroyed but can simply be transfered from one body to another.

⇒ **Quantisation of charge:** Net charge on a body is always an integral multiple of magnitude of charge on an electron.

$$\boxed{q = \pm ne}$$

where $n = 1, 2, 3, \ldots$

and e = +ve quantity equal to 1.6×10^{-19}

Coulomb's Law: The force of attraction or repulsion b/w two point charges is directly proportional to the product of charges and inversely proportional to the square of distance between them. The direction of this force is along the line joining the two charges. Thus

$$\vec{F} \propto \frac{q_1 q_2}{r^2} \Rightarrow \boxed{F = k\frac{q_1 q_2}{r^2}}$$

where $k = \frac{1}{4\pi\epsilon_0}$

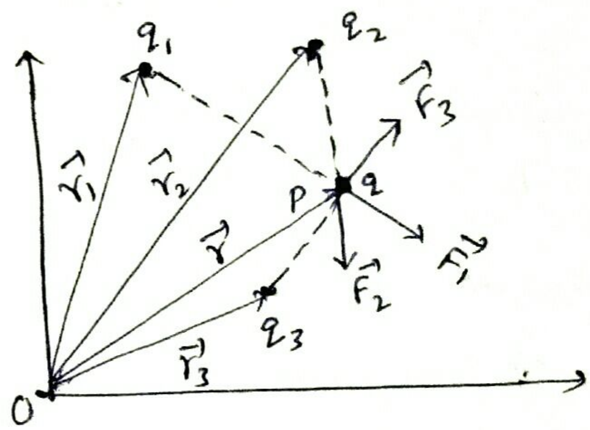
Force is a vector quantity

Principle of superposition: If the system containing a number of interacting charges, then the force on a given charge is equal to the vector sum of the forces exerted on it by all remaining charges.

Suppose a system contains n charges $q_1, q_2, q_3 \cdots q_n$ having position vectors $r_1, r_2, r_3 \cdots r_n$ relative to origin O respectively. A point charge q is placed at point P having position vector r relative to O.

The total force on charge q is

$$\vec{F} = \vec{F_1} + \vec{F_2} + \vec{F_3} + \cdots + \vec{F_n}$$



$$\therefore \vec{F_1} = \frac{1}{4\pi\epsilon_0} \frac{q q_1}{|\vec{r}-\vec{r_1}|} (\vec{r}-\vec{r_1})$$

$$\vec{F_2} = \frac{1}{4\pi\epsilon_0} \frac{q q_2}{|\vec{r}-\vec{r_2}|} (\vec{r}-\vec{r_2})$$

$$F_3 = \frac{1}{4\pi\epsilon_0} \frac{q q_3}{|\vec{r}-\vec{r_3}|} (\vec{r}-\vec{r_3})$$

$$\vec{F_n} = \frac{1}{4\pi\epsilon_0} \frac{q q_n}{|\vec{r}-\vec{r_n}|} (\vec{r}-\vec{r_n})$$

$$\therefore \boxed{F = \frac{q}{4\pi\epsilon_0} \sum_{i=1}^{n} \frac{q_i}{|\vec{r}-\vec{r_i}|} (\vec{r}-\vec{r_i})}$$

Electric field: The electric field at any point is defined as force per unit positive test charge.

$$\boxed{E = \lim_{q_0 \to 0} \frac{F}{q_0}}$$

This is a vector quantity

here q_0 is the +ve test charge

Electrostatic Potential : Electrostatic Potential is defined as the work done in bringing a unit positive charge from infinity to that point in the electric field.

$$\boxed{V = \frac{1}{4\pi\epsilon_0}\frac{q}{r}}$$

This is a scalar quantity

electrostatic Potential at a distance r.

Relation b/w $\vec{E}$ & V :

$$\vec{E}_x = -\frac{dV}{dx}\hat{i}, \quad \vec{E}_y = -\frac{\partial V}{\partial y}\hat{j}$$

$$\& \quad \vec{E}_z = -\frac{\partial V}{\partial z}\hat{k}$$

$$\because \quad \vec{E} = \vec{E}_x + \vec{E}_y + \vec{E}_z$$

$$\vec{E} = -\left(\hat{i}\frac{\partial}{\partial x} + \hat{j}\frac{\partial}{\partial y} + \hat{k}\frac{\partial}{\partial z}\right)V$$

$$\boxed{\vec{E} = -\vec{\nabla}V} \quad \text{or} \quad \boxed{\vec{E} = -\text{grad } V}$$

* ~~Electric field ($\vec{E}$) and Potential due to uniformly charged Sphere~~ :

~~Suppose a charge q is uniformly distributed over a sphere of radius R~~ ,

Poisson's and laplace's equation :

$$\because \quad \vec{E} = -\nabla V \quad \text{— (i)}$$

And from Gauss law in differential form

$$\text{div}.\vec{E} = \rho/\epsilon_0 \quad \text{— (ii)}$$

from (i) & (ii)

$$\text{div}.(-\nabla V) = \rho/\epsilon_0$$

Poisson's eqn.

(3) $$\nabla.(\nabla V) = -\rho/\epsilon_0 \Rightarrow \boxed{\nabla^2 V = \frac{-\rho}{\epsilon_0}} \quad \text{— (iii)}$$

and if we have a space where there is no free charge i.e. $\rho = 0$

so eq^n (iii) becomes

$$\boxed{\nabla^2 V = 0} \quad \text{(iv)}$$

This eq^n is known as Laplace's eq^n.

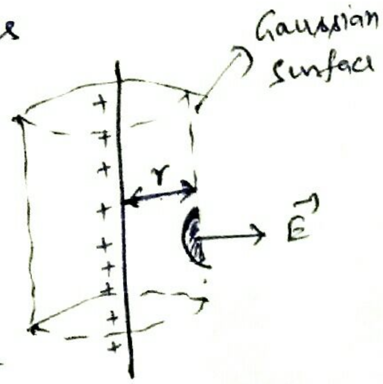
Gauss Law in electrostatics: It states that total electric flux through a closed surface is equal to $\frac{1}{\epsilon_0}$ times the net charge enclosed by the surface.

Mathematically

$$\boxed{\oint \vec{E} \cdot \vec{ds} = \frac{1}{\epsilon_0} q}$$

Applications of Gauss Law:

(1). Electric field due to infinite line charge



Suppose λ is line charge density

$\therefore$ $\boxed{\lambda = \frac{q}{l}}$ —— (1) $\Rightarrow$ $\boxed{q = \lambda l}$

here l is the length of the line.

According to Gauss Law

$$\oint \vec{E} \cdot \vec{ds} = \frac{q}{\epsilon_0}$$

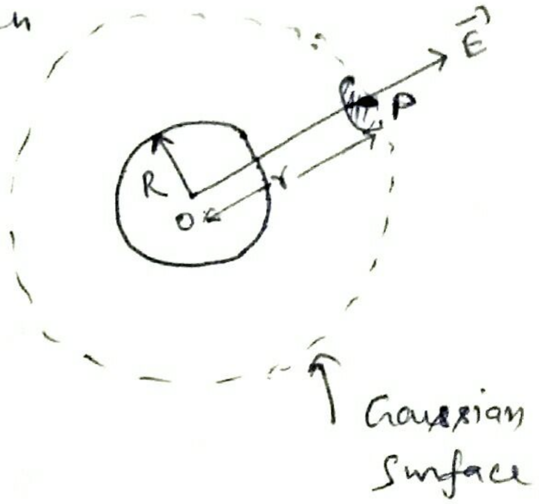
$$E \cdot 2\pi r \cancel{l} = \frac{\lambda \cancel{l}}{\epsilon_0}$$

$$\boxed{E = \frac{\lambda}{2\pi r \epsilon_0}}$$

② Electric field of a uniformly charged sphere:

Case 1: when point lies outside the sphere (External point)

Let O be the centre and R the radius of a sphere on which charge q has been distributed uniformly



Consider a Gaussian surface of radius r passing through the point P where we want to find out the electric field.

Apply Gauss law at point P.

$$\oint E \cdot ds = \frac{q}{\epsilon_0}$$

$$E \cdot 4\pi r^2 = \frac{q}{\epsilon_0} \Rightarrow \boxed{E = \frac{q}{4\pi\epsilon_0 r^2}}$$

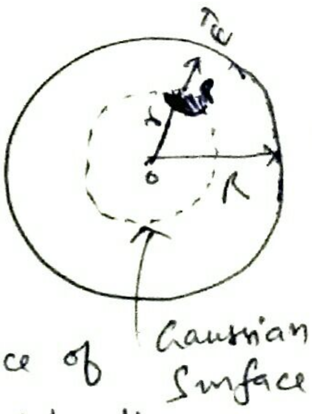
Case 2: when point P lies on the surface

In this case $r = R$

so $\boxed{E = \frac{q}{4\pi\epsilon_0 R^2}}$

Case 3: when point P lies inside the sphere (internal point)

Consider a Gaussian surface of radius r $(r<R)$



Total flux through the surface

$$\phi = \oint E \cdot ds = E \cdot 4\pi r^2 \quad \text{——} (1)$$

(i) If the sphere is conducting, the total charge will reside on the surface of sphere. Hence there is no charge inside the sphere. Hence $\boxed{E = 0}$

(ii) If sphere is non conducting :
There will be a distribution of charge throughout its volume. Let q' is the charge enclosed by Gaussian surface. And ρ is the charge density. Then

$$\rho = \frac{q}{\frac{4}{3}\pi R^3} \quad \text{—①}$$

$$q' = \rho \cdot \frac{4}{3}\pi r^3$$

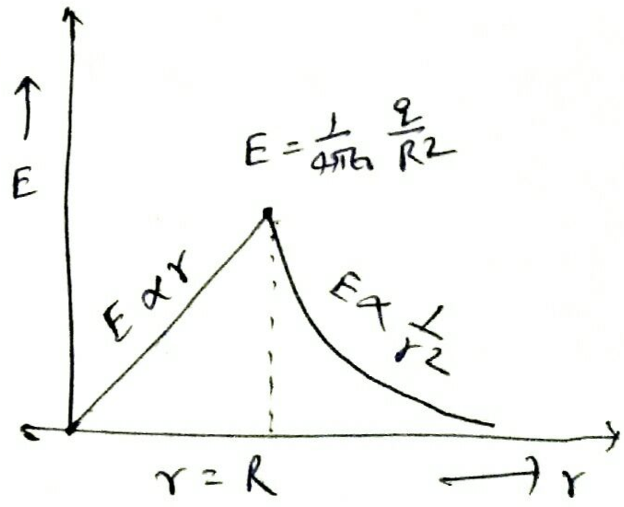
$$q' = \frac{q}{\cancel{\frac{4}{3}\pi} R^3} \cdot \cancel{\frac{4}{3}\pi} r^3 \Rightarrow \quad q' = q\left(\frac{r}{R}\right)^3$$

Now Apply Gauss Law at Point P.

$$\oint \vec{E} \cdot d\vec{s} = \frac{q'}{\varepsilon_0}$$

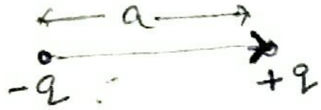
$$E \cdot 4\pi r^2 = \frac{q}{\varepsilon_0}\left(\frac{r}{R}\right)^3$$

$$\boxed{E = \frac{q}{4\pi\varepsilon_0 R^3}\, r}$$



Electric field due to an electric dipole.

Electric dipole: It is a system of two equal and opposite charges separated by a small distance.



Electric dipole moment ($\vec{p}$):

It is defined as the product of magnitude of either charge and separation b/w the two charges. It is a vector quantity and direction directed from -ve charge to +ve charge.

$$\boxed{\vec{p} = q \cdot \vec{a}}$$

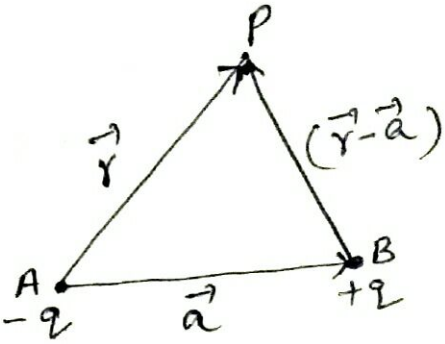


Fig. 1

In fig. 1. suppose P is a point ~~having~~ position vector $\vec{r}$ relative to A.

The electric field due to -q at Point P is

$$\vec{E}_1 = -\frac{1}{4\pi\epsilon_0}\frac{q\vec{r}}{r^3} \longleftarrow (i)$$

Similarly electric field due to charge +q at Point P is

$$\vec{E}_2 = \frac{1}{4\pi\epsilon_0}\frac{q(\vec{r}-\vec{a})}{|\vec{r}-\vec{a}|^3} \longrightarrow (ii)$$

∴ The electric field due to both +q & -q at Point P is

$$\vec{E}(r) = \vec{E}_1 + \vec{E}_2$$

$$\vec{E}(r) = \frac{q}{4\pi\epsilon_0}\left[\frac{(\vec{r}-\vec{a})}{|\vec{r}-\vec{a}|^3} - \frac{\vec{r}}{r^3}\right] \longleftarrow (iii)$$

Assuming $r >> a$, so

$$|\vec{r}-\vec{a}|^{-3} = \left[(\vec{r}-\vec{a})^2\right]^{-3/2}$$

$$|\vec{r}-\vec{a}|^{-3} = \left[r^2 - 2\vec{r}\cdot\vec{a} + a^2\right]^{-3/2}$$

$$|\vec{r}-\vec{a}|^{-3} = r^{-3}\left[1 - \frac{2\,\vec{r}\cdot\vec{a}}{r^2} + \underbrace{\frac{a^2}{r^2}}_{\simeq 0}\right]^{-3/2} \quad \because r >> a$$

$$\simeq r^{-3}\left[1 + \frac{3(\vec{r}\cdot\vec{a})}{r^2}\right]$$

$$|\vec{r}-\vec{a}|^{-3} \simeq \frac{1}{r^3}\left[1 + \frac{3(\vec{r}\cdot\vec{a})}{r^2}\right] \longleftarrow \text{(IV)}$$

Put the value of $|\vec{r}-\vec{a}|^{-3}$ from eqn (IV) into eqn (III) we get

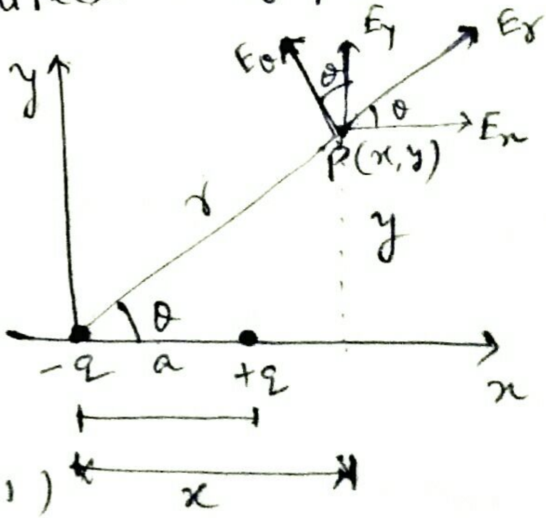
$$E(r) = \frac{q}{4\pi\epsilon_0}\left[(\vec{r}-\vec{a})\frac{1}{r^3}\left\{1 + \frac{3(\vec{r}\cdot\vec{a})}{r^2}\right\} - \frac{\vec{r}}{r^3}\right]$$

$$E(r) = \frac{q}{4\pi\epsilon_0}\left[\cancel{\frac{\vec{r}}{r^3}} + \frac{3\vec{r}(\vec{r}\cdot\vec{a})}{r^5} - \frac{\vec{a}}{r^3} - \underbrace{\frac{3\vec{a}(\vec{r}\cdot\vec{a})}{r^5}}_{\simeq 0} - \cancel{\frac{\vec{r}}{r^3}}\right]$$

$$E(r) = \frac{1}{4\pi\epsilon_0}\left[\frac{3\vec{r}(\vec{r}\cdot\vec{p})}{r^5} - \frac{\vec{p}}{r^3}\right] \text{ — (V)} \quad \because q\cdot\vec{a} = \vec{p}$$

Now we have to find the components of electric field at point P.

Suppose x-axis is along the direction of $\vec{p}$.



So

$$E_x = \frac{1}{4\pi\epsilon_0}\left[\frac{3px^2}{r^5} - \frac{p}{r^3}\right]$$

$$= \frac{1}{4\pi\epsilon_0}\left[\frac{3p}{r^3}\left(\frac{x}{r}\right)^2 - \frac{p}{r^3}\right]$$

$$\boxed{E_x = \frac{p}{4\pi\epsilon_0 r^3}\left[3\cos^2\theta - 1\right]} \longleftarrow \text{(VI)}$$

Similarly

$$E_y = \frac{1}{4\pi\epsilon_0}\left[\frac{3y(xp)}{r^5} - 0\right]$$

$$E_y = \frac{1}{4\pi\epsilon_0}\left[\frac{3p}{r^3}\left(\frac{x}{r}\right)\left(\frac{y}{r}\right)\right]$$

$$\boxed{E_y = \frac{3p}{4\pi\epsilon_0 r^3}\cos\theta\sin\theta}$$

$\because \left.\begin{array}{l}\frac{x}{r} = \cos\theta \\ \frac{y}{r} = \sin\theta\end{array}\right\}$

In Polar Coordinates : from eqn (V)

$$E_r = \frac{1}{4\pi\epsilon_0}\left(\frac{3(rp\cos\theta)r}{r^5} - \frac{p\cos\theta}{r^3}\right)$$

$\because \vec{r}\cdot\vec{p} = rp\cos\theta$

$\& \ \vec{p} = p\cos\theta\,\hat{r}$

$$\boxed{E_r = \frac{1}{4\pi\epsilon_0}\left[\frac{2p\cos\theta}{r^3}\right]}$$

Similarly

$$E_\theta = \frac{1}{4\pi\epsilon_0}\left(0 - \frac{p\sin\theta}{r^3}\right)$$

$$\boxed{E_\theta = \frac{p\sin\theta}{4\pi\epsilon_0 r^3}}$$